

# Index

- 3D reconstruction, 123
- 5+1-point algorithm, 272
- 5-point algorithm, 256
- 7-point algorithm, 251
- 8-point algorithm, 249
- affine point, 43
- affine transformation, 55
- affine transformation group, 55
- affine vector, 43
- algebraic constraint, 177
- algebraic distance, 38
- algebraic error, 38, 176
- algebraic representation, 147
- algebraic representation of  $SO(3)$ , 149
- algebraically independent, 117
- anti-Hermitian matrix, 329
- aperture, 93, 280
- associative operation, 334
- automatic camera calibration, 278
- auxiliary variables, 189, 214, 254, 291
- axis-angle representation, 150
- backward interpolation, 299
- barrel distortion, 283
- barycentric coordinates, 232
- baseline, 123
- blending, 300
- block diagonal matrix, 341
- block matrix, 341
- bounding box, 108, 294
- Brown-Conrady distortion, 285
- bundle adjustment, 315, 316
- C-normalized image coordinates, 98
- calibrated epipolar geometry, 121, 248
- calibration device, 288
- camera, 93
- camera calibration, 107, 208, 277
- camera center, 92
- camera centered coordinate systems, 94
- camera matrices from  $\mathbf{F}$ , 131
- camera matrix, 92
- camera normalized image coordinates, 98
- camera obscura, 93
- camera projection matrix, 92
- camera resectioning, 106
- canonical form, 28, 30, 63, 64, 68, 71
- canonical homogeneous coordinates, 88
- Cardano angles, 161
- Cartesian representation, 18
- Cayley transform, 155
- central perspective, 91
- central projection, 91
- centroid, 171
- $\mathbb{C}^n$ , 327
- co-linear planes, 22
- co-linear points, 18, 22
- co-planar lines, 22
- co-planar points, 21
- collineation, 83
- column compression, 346
- commutative operation, 334
- complex conjugate, 327
- complex conjugation, 328
- conjugate transpose, 329
- consensus set, 261
- consistent parameterization, 214
- consistent Plücker coordinates of 3D line, 70
- constraint enforcement, 215
  - fundamental matrix, 249
  - normalized camera matrix, 238
  - rotation matrix, 229
- corner points, 182
- correspondence, 121
- correspondence problem, 190, 191
- corresponding points, 121, 187
- cost function, 170
- cross product, 337
- cross product matrix, 337
- cross product operator, 337
- cross-correlation, 171
- D-normalization, 32
- data degeneracy, 193
- data error, 183
- data matrix, 34, 175
- decentering distortion, 285
- degeneracy
  - epipolar geometry estimation, 251
  - homography estimation, 193
- degenerate configuration, 50

degenerate data, 33  
 degrees of freedom, 49, 213  
 depth, 97  
 depth-of-field, 280  
 descent direction, 360  
 direct linear transformation, 117, 189  
 directed line, 39  
 direction, 43  
 distributive operation, 334  
 distortion center, 286  
 DL-normalization, 71  
 DLT, 189  
 double embedding of  $SO(3)$ , 158  
 dual homogeneous coordinates, 30, 65  
 dual line normalization, 32, 71  
 dual Plücker coordinates, 47, 69  
 dual transformation, 60  
 duality mapping, 70  
 duality principle, 47  
 Eckart-Young (-Mirsky) theorem, 349  
 ego-motion estimation, 316  
 energy function, 170  
 epipolar constraint, 117, 121, 127  
 epipolar geometry, 121
 

- degenerate, 251

 epipolar homographies, 126  
 epipolar line, 124, 128  
 epipolar plane, 124  
 epipolar points, 123  
 epipoles, 123  
 equation of the line, 18  
 equation of the plane, 21  
 equilibrating transformation, 203  
 equivalent cameras, 112  
 error function, 170  
 essential matrix, 121, 139, 140
 

- estimation
  - robust, 270
  - internal constraint, 140

 estimation
 

- camera matrix, 206
- epipolar geometry, 241
- fundamental matrix, 248
- homography, 187
- introduction, 169
- rigid transformation, 231
- rotations, 227
- transformations, 187

 Euclidean reconstruction, 135  
 Euler angles, 161  
 exterior orientation, 104, 231  
 external camera parameters, 104  
 FET, 284  
 field of view, 118  
 field of view model, 284  
 fish-eye lens, 280  
 fish-eye transform, 284  
 focal length, 94  
 focal point, 92  
 forward interpolation, 299  
 FOV, 284  
 Frobenius norm, 330  
 Frobenius scalar (or inner) product, 330  
 full rank pseudo-inverse, 339  
 fundamental matrix, 121, 127
 

- camera matrices from, 131
- estimation, 248
  - robust, 269
  - internal constraint, 127

 Gauss-Newton method, 364  
 general configuration, 50  
 geometric distortion, 280  
 geometric objects, 27  
 gimbal lock, 165  
 gold standard method, 254  
 group
 

- examples
  - special unitary group, 330
  - $SU(n)$ , 330
  - $U(n)$ , 330
  - unitary group, 330

 H-normalization, 203  
 hand-eye calibration, 231  
 Hartley normalization, 203  
 Hermitean
 

- matrix, 329
- scalar product, 327

 homogeneous coordinates, 25, 28, 63  
 homogeneous method, 179  
 homogeneous representation, 18  
 homography, 83  
 homography estimation
 

- degeneracy, 193
- robust, 269

 homography transformation, 83  
 hybrid method, 364  
 ICP, 233  
 ideal data, 183  
 ideal point, 43  
 identical noise, 184  
 identity element, 334  
 identity rotation, 150  
 image coordinate system, 101  
 image coordinates, 92  
 image mosaic, 294

image mosaicking, 294  
 image plane, 92  
 image point, 92  
 incidence relations, 32  
 independent noise, 184  
 indeterministic estimation method, 261  
 inhomogeneous method, 177  
 inliers, 258  
 inner derivatives, 216  
 intercept, 20  
 interior orientation, 104  
 internal camera parameters, 104  
 internal constraint  
     essential matrix, 140  
     fundamental matrix, 127  
     normalized camera matrix, 99  
 Plücker coordinates, 70  
     rotation matrix, 148  
 invariances, 49  
 inverse of a quaternion, 335  
 involution, 58  
 isotropic noise, 184  
 iterative closest point, 233  
  
 Jacobian, 363  
 Jacobian mask, 322, 368  
  
 keystone effect, 86  
  
 L-normalization, 68  
 $L_1$  error, 175  
 $L_2$  error, 175  
 left inverse, 339  
 left singular vectors, 343  
 lens distortion, 280  
 lens distortion function, 281  
 lens effect, 280  
 Levenberg-Marquardt method, 364  
 line at infinity, 44, 67  
 line normalization, 68  
 linear transformation, 332  
  
 matrix exponential function, 352  
 matrix logarithm, 353  
 maximum likelihood estimation, 184  
 mean, 171  
 measurement error, 183  
 measurement noise, 110  
 method degeneracies, 247  
 method degeneracy, 193  
 mid-point method, 242  
 minimal case estimation, 192  
 minimal parameterization, 104, 130  
 minimal representation, 149  
 ML-estimation, 184  
  
 model error, 180  
 Moore-Penrose inverses, 340  
 motion stereo, 123  
  
 nearest neighbor interpolation, 111, 299  
 Newton's method, 362  
 non-linear optimization, 359  
 norm  
     Frobenius, 330  
 normalized 8-point algorithm, 250  
 normalized camera, 97  
 normalized camera matrix, 97, 99  
     internal constraint, 99  
 normalizing transformation, 200, 203  
  
 objective function, 170  
 observation, 183  
 omni-directional camera, 293  
 one-first enumeration, 110  
 OPP, 347  
 optical axis, 94  
 optical center, 92  
 optical ray, 111  
 optimal triangulation, 243  
 orientation of a line, 42  
 orthogonal Procrustes problem, 347  
 outliers, 258  
 over-fitting, 181  
  
 P-normalization, 29  
 P3P, 240  
 panorama image, 294  
 panorama stitching, 294  
 parameter space, 149  
 parameterization of a transformation, 50  
 perspective  $n$ -point problem, 237  
 pin-cushion distortion, 283  
 pinhole camera, 91, 93  
 pinhole perspective, 91  
 pixel coordinates, 101  
 pixel density, 105  
 pixel grid, 108  
 pixel value, 109  
 pixels, 93  
 Plücker coordinates of 2D line, 46  
 Plücker coordinates of 3D line, 67  
     consistent, 70  
     internal constraint, 70  
 plumb bob distortion, 285  
 PnP, 237  
 point at infinity, 42  
 point cloud, 320  
 point normalization, 29  
 points of interest, 182, 191  
 pose, 104

pre-conditioning, 203  
 principal axis, 94  
 principal line, 94  
 principal plane, 94  
 principal point, 94  
 projection, 83  
 projection line, 83, 111  
 projection plane, 116  
 projection point, 83  
 projective elements, 331  
 projective reconstruction, 135  
 projective space, 331  
 projectivity, 83  
 proper line, 67  
 proper lines, 41  
 proper points, 41  
 pseudo-inverse, 339
 

- full rank, 339

 pure quaternion, 335  
 putative correspondences, 312  
 quaternion, 334
 

- inverse, 335
- pure, 335

 quaternion sandwich product, 157  
 quaternionic embedding of  $\mathbb{R}^3$ , 157  
 radial distortion function, 282  
 Random Sample Consensus, 262  
 RANSAC, 262  
 re-mapping of the residual vector, 217  
 re-normalize, 199  
 re-parameterization, 215  
 reconstruction, 134  
 rectified stereo rig, 123, 136  
 rectifying homographies, 138  
 reference image, 295  
 registration, 233  
 representative of a projective element, 331  
 reprojection error, 315  
 residual, 363  
 residual error, 171  
 residual vector, 212  
 right inverse, 339  
 right singular vectors, 343  
 rigid transformation, 53
 

- estimation, 231

 rigid transformation group, 53  
 robust errors, 259  
 robust estimation, 257, 258
 

- essential matrix, 270
- fundamental matrix, 269
- homography, 269

 Rodrigues' rotation formula, 150, 152  
 rotation, 148  
 rotation matrix
 

- internal constraint, 148

 row compression, 346  
 SAD, 175  
 sandwich product, 82  
 scalar product
 

- Frobenius, 330

 Schur complement, 342  
 Schur complement trick, 324  
 $SE(2)$ , 53  
 $SE(3)$ , 79  
 semi-linear, 328  
 sesqui-linear, 328  
 SfM, 312  
 shearing, 58  
 signed distance, 37  
 similarity reconstruction, 135  
 similarity transformations, 54  
 singular value decomposition, 342  
 singular values, 343  
 skewing, 58  
 slope, 20  
 solution space, 193  
 SOPP, 229  
 special Euclidean transformation group
 

- in  $\mathbb{E}^2$ , 53
- in  $\mathbb{E}^3$ , 79

 special orthogonal Procrustes problem, 229  
 special QR-factorization, 358  
 special SVD, 348  
 special unitary group, 330  
 SSD, 175  
 SSVD, 348  
 standard deviation, 184  
 standard Plücker coordinates, 69  
 step length, 360  
 stereo camera, 121  
 stereo rectification, 123  
 stereo rig, 121, 123  
 structure from motion, 312  
 $SU(n)$ , 330  
 $su(n)$ , 329  
 SVD, 342  
 SVD profile, 195  
 Tait-Bryan angles, 161  
 thin-prism distortion, 285  
 three-angle representation, 161  
 total least squares, 172  
 translation group, 52  
 trial set, 261  
 triangulation, 117, 123, 134  
 twisted pair, 162  
 twisted rotations, 142

$U(n)$ , 330

unbiased estimate, 205

unbiased noise, 184

uncalibrated epipolar geometry, 121, 131

uniform noise, 184

unit quaternion, 335

unitary group, 330

unitary matrix, 330

unperturbed data, 183

variances, 171

vector product, 337

vectorization, 190, 207

virtual image plane, 97

visibility function, 315

visual appearance, 267

Wahba's problem, 228

world coordinate system, 98

zero-first enumeration, 110